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The nucleon mass and pion-nucleon sigma term from a chiral analysis of $N_f = 2 + 1$ lattice QCD world data

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Fits of the p^4 covariant $SU(2)$ baryon chiral perturbation theory to lattice QCD nucleon mass data from several collaborations for 2 and 2+1 flavors are presented. We consider contributions from explicit Δ (1232) degrees of freedom, finite volume and finite spacing corrections. We emphasize here on our $N_f = 2 + 1$ study. We obtain low-energy constants of natural size that are compatible with the rather linear pion-mass dependence of the nucleon mass observed in lattice QCD. We report a value of $\sigma_{\pi N} = 41(5)(4)$ MeV in the 2 flavor case and $\sigma_{\pi N} = 52(3)(8)$ MeV for 2+1 flavors.

1. Introduction

Through lattice QCD simulations (lQCD) it is possible to study QCD at current quark masses which are not restricted to their physical values. Data points obtained for such a scenario represent a powerful pool of information to fix free low-energy constants (LECs) of the (baryon) chiral perturbation theory (B χ PT); LECs that also enter calculations for experimental observables. Thus, adjusting the B χ PT to lQCD data in the unphysical region allows for predictions at the physical point.

In our study we performed such a matching for the quark-mass ($m_u = m_d = \overline{m}$) dependence of the nucleon mass $M_N(\overline{m})$. We fit lQCD data for the $N_f = 2$ flavors, two light degenerated quarks, as well as for the $N_f = 2 + 1$ ensembles, two light degenerated and one heavy quarks. Here, we concentrate on our $N_f = 2 + 1$ results and refer to Ref. ¹ for more details on the $N_f = 2$ ones.

A measure of the contribution from the explicit chiral symmetry breaking to the nucleon mass is given by the so-called $\sigma_{\pi N}$ -term. It is the nucleon scalar form factor at zero momentum transfer squared which can be isolated from πN -scattering experiments. Alternatively, the $\sigma_{\pi N}$ can also be obtained by the Hellmann-Feynman theorem from $M_N(\overline{m})$:

$$\overline{m} \frac{\partial}{\partial \overline{m}} M_N(\overline{m}) = \sigma_{\pi N} = \overline{m} \langle N | \overline{u}u + \overline{d}d | N \rangle . \quad (1)$$

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As a result, the function $M_N(\overline{m})$, which has mainly unphysical values, connects lQCD, B χ PT and experiment.

2. Nucleon mass and covariant baryon chiral perturbation theory

The B χ PT up to p^4 describes the nucleon mass by:

$$M_N^{(4)}(M_\pi^2) = M_0 - c_1 4M_\pi^2 + \overline{\alpha} \frac{M_\pi^4}{2} + \frac{c_1 M_\pi^4}{8\pi^2 f_\pi^2} \ln \frac{M_\pi^2}{M_0^2} + \Sigma_{loops}^{(3+4)}(M_\pi^2) + \mathcal{O}(p^5), \quad (2)$$

with $M_\pi^2 \sim \overline{m}$ and f_π as the pion mass and pion decay constant. We fit the three LECs c_1 , $\overline{\alpha}$ and M_0 to lQCD data and obtain through Eq. (1) a $\sigma_{\pi N}$ value. Explicit $\Delta(1232)$ contributions can appear in the loop-contributions $\Sigma_{loops}^{(3+4)}$ and we refer to Ref. ¹ for all technical details. Generally, lQCD data is given in the dimensionless form (aM_π, aM_N) where each collaboration sets its scale through a specific value for the lattice spacing a . In the $N_f = 2 + 1$ case, we fit the data after converting to physical units. Since all three quantities are subject to statistical uncertainties, the a uncertainty translates into a correlated uncertainty for the normalization of data points belonging to the same set. We take this correlation into account by defining a correlation matrix V containing all these uncertainties, and minimize the function:

$$\chi^2 = \vec{\Delta}^T V^{-1} \vec{\Delta}, \quad \Delta_i = M_N^{(4)}(M_{\pi,i}^2) + c_i a_i^2 + \Sigma_N^{(4)}(M_{\pi,i}^2, L_i) - d_i(M_{\pi,i}^2, L_i), \quad (3)$$

with d_i being the data and $M_N^{(4)}$ and $\Sigma^{(4)}$ the B χ PT infinite volume nucleon mass and its finite volume corrections. The constants c_i parametrize finite spacings effects for each lQCD-action separately. In our $SU(2)$ B χ PT analysis we only include $N_f = 2 + 1$ lQCD data for which the strange quark mass is kept constant approximately at its phenomenological value. For this case the strange quark mass corrections to the nucleon mass can be seen as being integrated out into the LECs we are fitting. Furthermore, to ensure controllable finite volume corrections and a reasonable chiral convergence of our results, we only account for data points with $M_\pi L > 3.8$ and $M_\pi < 415$ MeV, with L being the lattice box size. Following this, we include data from the BMW ², HSC ³, LHPC ⁴, MILC ⁵, NPLQCD ⁶, PACS ⁷ and RBCUK-QCD ⁸ collaborations.

3. Results and conclusion

The Fig. 1 shows several B χ PT fit strategies to the above listed $N_f = 2 + 1$ lQCD data. We obtain consistent descriptions of the pion-mass dependence of the nucleon mass where the slope-variations are below the resolution of the current data. However, by looking to the pion-mass dependence of the $\sigma_{\pi N}$ term, left graph of Fig. 2, we observe that these small variations translate into noticeable changes. For successively including explicit $\Delta(1232)$ contributions and finite spacing corrections in our fit formula, the $\sigma_{\pi N}$ term at the physical point changes by up to 14 MeV from $\sigma_{\pi N} = 58(3)$ MeV to 49(2) MeV down to 44(3) MeV. In view of this, we take a conservative standpoint and give the following weighted average as our final $N_f = 2 + 1$

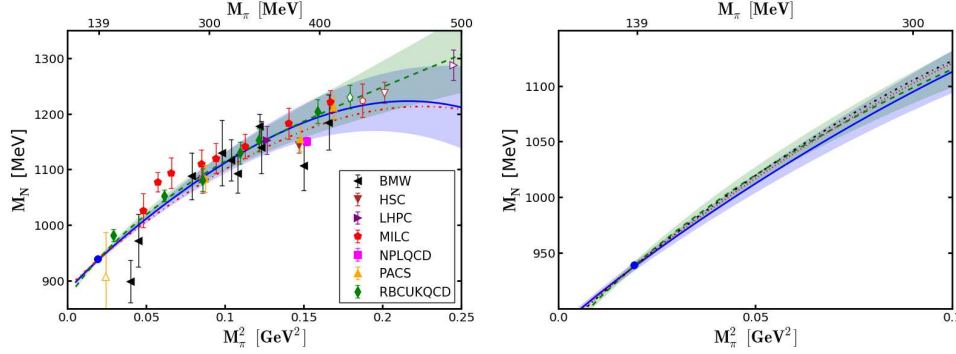


Fig. 1. Pion-mass dependence of the nucleon mass from $N_f = 2 + 1$ fits. Right: The blue-solid (green-dashed) line shows our result for the $\mathcal{O}(p^4)$ B χ PT fit with(out) $\Delta(1232)$ contributions by excluding the two data points from the HSC and NPLQCD collaborations. The red-dotted line is a B χ PT fit with $\Delta(1232)$ including also this data. Left: Same results depicted for a smaller M_π range.

$\sigma_{\pi N}$ value at the physical point:

$$\sigma_{\pi N}^{N_f=2+1} = 52 (3) (8) \text{ MeV} , \quad (4)$$

where the second (systematical) uncertainty spans all the above central values.

The right graph of Fig. 2 compares our B χ PT results from separate fits to lQCD data from $N_f = 2$ and $N_f = 2 + 1$ ensembles. Also here, differences are within the spread and uncertainties of the input data but result in a slightly smaller $\sigma_{\pi N}$ value for the $N_f = 2$ case:

$$\sigma_{\pi N}^{N_f=2} = 41 (5) (4) \text{ MeV} . \quad (5)$$

Concerning the uncertainties, these two values are compatible, however, show a slight tension which we mostly trace back to the different data point distributions. On the one hand, the current $N_f = 2$ data does not constrain the low- M_π region as much as the $N_f = 2 + 1$ data. This region is dominated by the LEC c_1 which is the $\sigma_{\pi N}$ at leading order. Consequently, we obtain by ~ 13 % different c_1 -values with correspondingly different contributions to the total $\sigma_{\pi N}$. On the other hand, in the $N_f = 2$ case one direct data point⁹ of the $\sigma_{\pi N}$ at $M_\pi \approx 290$ MeV could be included in the fit. A positive outcome was that the above spread of our results with and without inclusion of $\Delta(1232)$ contributions is lifted. Since no such data point at low- M_π is available for the $N_f = 2 + 1$ case, a spread of 9 MeV remains. Note that in the HB χ PT formalism¹⁰ this spread is more than 40 MeV. To compare our above numbers also with phenomenology, we cite the latest value extracted from pure π -N scattering data: $\sigma_{\pi N} = 59(7)$ MeV¹¹. In conclusion, we performed fits of the p^4 $SU(2)$ covariant B χ PT with and without explicit $\Delta(1232)$ contributions to lQCD nucleon mass data. Even though the present data set is extensive, we spotted systematic effects coming mainly from the distribution of the data points.

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These systematic effects would further be reduced by new data for: a) more $N_f = 2$ data points in the low- M_π region, b) already one direct calculation of the $\sigma_{\pi N}$ at $M_\pi < 300$ MeV for the $N_f = 2 + 1$ case.

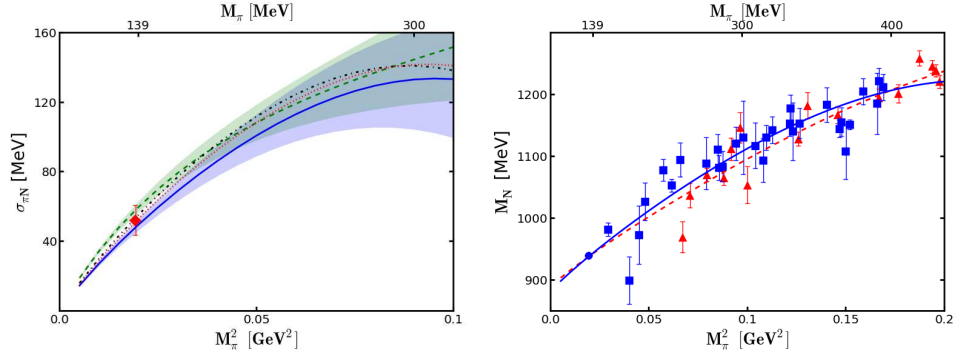


Fig. 2. Left: Pion-mass dependence of the $\sigma_{\pi N}$ term from $N_f = 2 + 1$ fits. The color code is the same as in Fig. 1. The red-diamond is our predicted $\sigma_{\pi N}$ value at the physical point. Right: Comparison of B χ PT p^4 fits with $\Delta(1232)$ to $N_f = 2$ (red-triangles) and $N_f = 2 + 1$ (blue-squares) IQCD data. The blue-circle is the physical nucleon mass.

Acknowledgments

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